Soft gluon multiplicity distribution revisited

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In this paper the soft gluon radiation from partonic interaction of the type: $2 \to 2$ + gluon has been revisited and a correction term to the widely used Gunion-Bertsch (GB) formula is obtained.

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The energy loss of high energy partons propagating through a thermalized system of quarks and gluons created in heavy ion collisions at relativistic energies has been measured through nuclear suppression factors at Relativistic Heavy Ion Collider (RHIC) energy [1]. The two most important mechanisms for the energy loss are radiative and collisional processes. Therefore, it is very important to understand and theoretically improve the calculations of partonic energy loss in thermal medium. Generically the radiative energy loss can be written as $2 \to 2+g$ process, here we consider the process $gg \to gg+g$, and the results for other process can be obtained from it in a straight forward way. The typical Feynman diagram for $gg \to gg+g$ is shown in Fig. 1. The momentum k_5 of the radiative gluon (Fig. 1) is

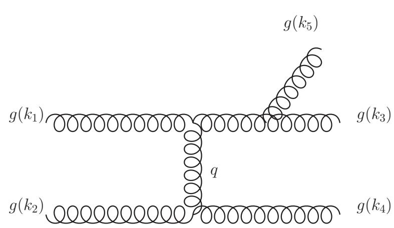

FIG. 1: A typical Feynman diagram for the process: $gg \to ggg$

taken to be a soft radiation around zero rapidity in the centre of momentum frame. The invariant amplitude for this process in medium is given by (see the appendix):

$$|M_{gg\to ggg}|^2 = \left(\frac{4g^4N_c^2}{N_c^2-1}\frac{s^2}{(q_\perp^2+m_D^2)^2}\right)\left(\frac{4g^2N_cq_\perp^2}{k_\perp^2[(k_\perp-q_\perp)^2+m_D^2]}\right) + \frac{16g^6N_c^3}{N_c^2-1}\frac{q_\perp^2}{k_\perp^2[(k_\perp-q_\perp)^2+m_D^2]} \qquad (1)$$

where $k_\perp$ and $q_\perp$ are the perpendicular component of k_5 and that of the momentum transfer in the centre of momentum frame respectively, m_D is the Debye mass, N_c is the number of colour degrees of freedom. The first parenthesis in the first term in Eq. 1 stands for the square of the invariant amplitude for the process: $gg \to gg$

$$|M_{gg\to gg}|^2 = \left(\frac{4g^4N_c^2}{N_c^2-1}\frac{s^2}{(q_\perp^2+m_D^2)^2}\right) \qquad (2)$$

and the second parenthesis in the first term represents the soft gluon emission spectrum [3]. The second term of Eq. 1 is the correction to the squared modulus of the matrix elements. The first term of Eq. 1 is reported in Ref. [3]. In the limit $m_D \to 0$ the first term also reproduces the results obtained in Ref. [2].

The soft gluon multiplicity distribution at fixed $q_\perp$ is given by [2, 3]:

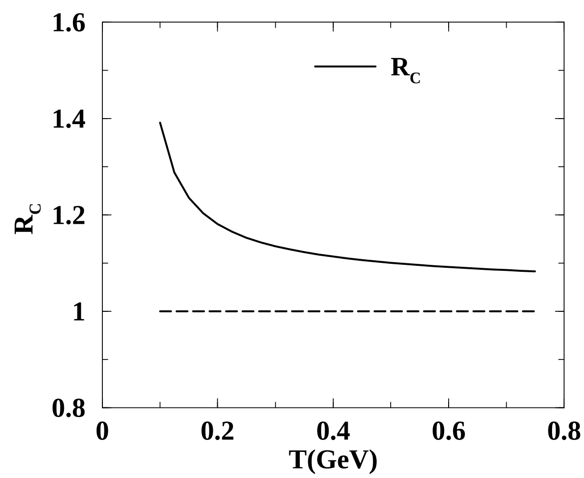

FIG. 2: The variation of R_c (see Eq. 6) with temperature.

$$\frac{dn_g}{d\eta dk_\perp^2} = \frac{C_A\alpha_s}{\pi^2}\left(\frac{q_\perp^2}{k_\perp^2[(k_\perp-q_\perp)^2+m_D^2]}\right) + \frac{C_A\alpha_s}{\pi^2}\left(\frac{q_\perp^2(q_\perp^2+m_D^2)^2}{s^2k_\perp^2[(k_\perp-q_\perp)^2+m_D^2]}\right) \qquad (3)$$

In Eq. 3 the second term is the correction to the soft gluon multiplicity distribution. Eq. 3 can be written as:

$$\frac{dn_g}{d\eta dk_\perp^2} = \left[\frac{dn_g}{d\eta dk_\perp^2}\right]_{GB}\left(1+\frac{(q_\perp^2+m_D^2)^2}{s^2}\right) \qquad (4)$$

where

$$\left[\frac{dn_g}{d\eta dk_\perp^2}\right]_{GB} = \frac{C_A\alpha_s}{\pi^2}\frac{q_\perp^2}{k_\perp^2[(k_\perp - q_\perp)^2 + m_D^2]} \quad (5)$$

To estimate the contributions from the correction term we consider the ratio, R_c given by

$$R_c = \frac{\frac{dn_g}{d\eta dk_\perp^2}}{\left[\frac{dn_g}{d\eta dk_\perp^2}\right]_{GB}} = 1 + \frac{(q_\perp^2 + m_D^2)^2}{s^2} \quad (6)$$

We evaluate R_c by substituting $s =< s >= 18T^2$, $m_D = \sqrt{4\pi\alpha_s(T)}\,T$ and $q_\perp^2 =< q_\perp^2 >$ which is calculated by using the following relation:

$$< q_\perp^2 >= \frac{\int dt\, t\,(d\sigma/dt)}{\int dt\,(d\sigma/dt)} \quad (7)$$

The lower and upper limits of the above integration are $= m_D^2$ and $s/4$ respectively. The variation of R_c with T is depicted in Fig. 2. The temperature dependence of the strong coupling α_s is taken from [5]. It is observed that the correction to the gluon spectrum is appreciable for low temperature domain.

The similar correction in the soft gluon multiplicity distribution can be obtained for the processes $qg \to qgg$ and $qq \to qqg$. This result can also be used for radiative energy loss of heavy quarks due to gluon emission. The effects of quark mass can be taken into account by multiplying the emitted gluon distribution from massless quarks by a factor, F^2 which takes in to account the dead cone effects [4]:

$$F = \frac{k_\perp^2}{\omega^2\theta_0^2 + k_\perp^2} \quad (8)$$

where $\theta_0 = M/E$, M is the mass and E is the energy of the heavy quarks. The results for the gluon spectrum emitted by a heavy quark of mass M can be obtained by multiplying the Eq. 4 by F^2. In summary we have derived an expression for the soft gluon multiplicity distribution from a process of the type: $2 \to 2 + g$. We observe that the corrections to the gluon spectrum obtained by [2, 3] is non-negligible in the low temperature region. This result will be useful in estimating the energy loss of high energy partons propagating through a thermalized system of quarks and gluons.

I. APPENDIX

In this appendix we outline the derivation of Eq. 1. The invariant amplitude for the process, $gg \to ggg$ can be written as [6]:

$$\begin{aligned}|M_{gg\to ggg}|^2 = \frac{1}{2}g^6\frac{N_c^3}{N_c^2-1}[&(12345) + (12354)\\ &+(12435) + (12453) + (12534) + (12543)\\ &+(13245) + (13254) + (13425) + (13524)\\ &+(14235) + (14325)]\frac{N}{D}\end{aligned} \quad (9)$$

where

$$\begin{aligned}N = &(k_1k_2)^4 + (k_1k_3)^4 + (k_1k_4)^4 + (k_1k_5)^4\\ &+(k_2k_3)^4 + (k_2k_4)^4 + (k_2k_5)^4 + (k_3k_4)^4\\ &+(k_3k_5)^4 + (k_4k_5)^4,\end{aligned} \quad (10)$$

$$\begin{aligned}D = &k_1.k_2k_1.k_3k_1.k_4k_1.k_5k_2.k_3k_2.k_4\\ &k_2.k_5k_3.k_4k_3.k_5k_4.k_5\end{aligned} \quad (11)$$

and

$$(ijklm) = (k_i.k_j)(k_j.k_k)(k_k.k_l)(k_l.k_m)(k_m.k_i) \quad (12)$$

Defining $s = (k_1 + k_2)^2, t = (k_1 - k_3)^2, u = (k_1 - k_4)^2$ and $s' = (k_3 + k_4)^2, t' = (k_2 - k_4)^2, u' = (k_2 - k_3)^2$, we can write $k_1.k_2 = s/2, k_3.k_4 = s'/2, k_1.k_3 = -t/2, k_2.k_4 = -t'/2, k_1.k_4 = -u/2, k_2.k_3 = -u'/2$

Eq. 9 contains twelve terms. Using Eqs. 10, 11 and Eq. 12 the first two terms of Eq. 9 can be expressed as:

$$\frac{1}{2}g^6\frac{N_c^3}{N_c^2-1}\frac{N}{k_1.k_3k_1.k_4k_2.k_4k_2.k_5k_3.k_5} \quad (13)$$

and

$$\frac{1}{2}g^6\frac{N_c^3}{N_c^2-1}\frac{N}{k_1.k_3k_1.k_5k_2.k_4k_2.k_5k_3.k_4} \quad (14)$$

respectively. All other terms in Eq. 9 can be reduced in the above form by following similar procedure.

The quantity N can be written as:

$$\begin{aligned}N = &\frac{1}{16}(s^4 + t^4 + u^4 + s\prime^4 + t\prime^4 + u\prime^4)\\ &+\sum_{i=1}^{4}(k_i.k_5)^4\end{aligned} \quad (15)$$

In the infrared and small angle scattering limits, we have : $k_5 \to 0, t' \to t, s' \to s, u' \to u,\ s \to -u$.

Using these approximations we get,

$$|M_{gg\to ggg}|^2 = g^6\frac{N_c^3}{N_c^2-1}s^4[\frac{1}{st^2k_2.k_5k_3.k_5} + \frac{1}{st^2k_1.k_5k_2.k_5} + \frac{1}{st^2k_1.k_5k_4.k_5} + \frac{1}{st^2k_3.k_5k_4.k_5}] + g^6\frac{N_c^3}{N_c^2-1}s^4[\frac{1}{s^3k_1.k_5k_2.k_5} - \frac{1}{s^2tk_2.k_5k_4.k_5} - \frac{1}{s^2tk_1.k_5k_3.k_5} + \frac{1}{s^3k_2.k_5k_3.k_5} - \frac{1}{s^2tk_1.k_5k_3.k_5} + \frac{1}{s^3k_4.k_5k_3.k_5} + \frac{1}{s^3k_1.k_5k_4.k_5} - \frac{1}{s^2tk_2.k_5k_4.k_5}] \quad (16)$$

The above equation may be simplified to obtain:

$$|M_{gg\to ggg}|^2 = 4g^6\frac{N_c^3}{N_c^2-1}\frac{s^3}{t^2}\frac{1}{k_1.k_5k_2.k_5} + 2g^6\frac{N_c^3}{N_c^2-1}[\frac{2s}{k_1.k_5k_2.k_5} - \frac{s^2}{t(k_2.k_5)^2} - \frac{s^2}{t(k_1.k_5)^2}] \quad (17)$$

The first term of the above equation is given by

$$\begin{aligned}|M_{gg\to ggg}|^2_{1st} &= 4g^4\frac{N_c^2}{N_c^2-1}\frac{s^2}{t^2}\frac{g^2N_cs}{k_1.k_5k_2.k_5} \\ &= |M_{gg\to gg}|^2\frac{g^2N_cs}{k_1.k_5k_2.k_5} \qquad (18)\end{aligned}$$

Using the following relations [3],

$$k_\perp^2 = 4k_1.k_5k_2.k_5/s, \qquad (19)$$

$$q_\perp^2 = 4k_1.k_4k_2.k_4/s, \qquad (20)$$

and

$$(k_\perp - q_\perp)^2 = 4k_1.k_3k_2.k_3/s, \qquad (21)$$

We get,

$$\frac{4g^2N_cq_\perp^2}{k_\perp^2[(k_\perp-q_\perp)^2+m_D^2]} = \frac{g^2N_cs}{k_1.k_5k_2.k_5} \qquad (22)$$

and substituting Eq.22 in Eq. 18 we obtain,

$$|M_{gg\to ggg}|^2_{1st} = \left(\frac{4g^4N_c^2}{N_c^2-1}\frac{s^2}{(q_\perp^2+m_D^2)^2}\right)\left(\frac{4g^2N_cq_\perp^2}{k_\perp^2[(k_\perp-q_\perp)^2+m_D^2]}\right) \qquad (23)$$

This is the result reported in Ref. [3]. Now we focus on the correction to this result i.e the remaining term in $|M_{gg\to ggg}|^2$, which is given by

$$|M_{gg\to ggg}|^2_{2nd} = 2g^6\frac{N_c^3}{N_c^2-1}[\frac{2s}{k_1.k_5k_2.k_5} - \frac{s^2}{t(k_2.k_5)^2} - \frac{s^2}{t(k_1.k_5)^2}] \qquad (24)$$

Using Eq. 19 we have

$$|M_{gg\to ggg}|^2_{2nd} = 2g^6\frac{N_c^3}{N_c^2-1}[\frac{2s}{k_1.k_5k_2.k_5} - \frac{16(k_1.k_5)^2}{tk_\perp^4} - \frac{16(k_2.k_5)^2}{tk_\perp^4}] \qquad (25)$$

In the limit $k_5 \to 0$, we have

$$\begin{aligned}|M_{gg\to ggg}|^2_{2nd} &= 2g^6\frac{N_c^3}{N_c^2-1}[\frac{2s}{k_1.k_5k_2.k_5}] \\ &= \frac{g^2N_cs}{k_1.k_5k_2.k_5}\frac{4g^4N_c^2}{N_c^2-1} \\ &= \frac{16g^6N_c^3}{N_c^2-1}\frac{q_\perp^2}{k_\perp^2[(k_\perp-q_\perp)^2+m_D^2]} \qquad (26)\end{aligned}$$

This is the correction term to the results obtained in [2, 3] for the process $gg \to 2 + g$.

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[1] J. Adams *et al.* (STAR Collaboration), Nucl. Phys. A **757**, 102 (2005); K. Adcox *et al.* (PHENIX Collaboration), Nucl. Phys. A **757**, 184,(2005). B. I. Abeleb *et al.* (STAR Collaboration), Phys. Rev. Lett. **98**, 192301 (2007); S. S. Adler *et al.* (PHENIX Collaboration), Phys. Rev. Lett. **96**, 032301 (2006).
[2] J. F. Gunion and G. Bertsch, Phys. rev. D **25**,746(1982)
[3] S. M. H. Wong, Nucl. Phys. A **607**,442(1996)
[4] Y. L. Dokshitzer and D. E. Kharzeev, Phys. Lett. B, **519**, 199 (2001).
[5] O. Kaczmarek and F. Zantow, Phys. Rev. D, **71**, 114510(2005)
[6] F. A. Berends *et al.*, Phys. Lett. B **103**, 124 (1981).